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Directional Derivative in Edge Detection

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Abstract— Step edges are localized as maxima of the gradient modulus taken in the direction of the gradient, or as zero-crossings of the Laplacian or the second directional derivative along the gradient direction. This paper shows that the step edge can be localized as a zero or positive minimum of the magnitude of the second derivative along the gradient direction, or the negative minimum of the third derivative. The results of applying the proposed methods to real images are given. The advantages and disadvantages of these methods over the classical methods are presented.

Index Terms— Edge detection, Phantom edges, and Authentic edges.

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1 Introduction

An edge is characterized by an abrupt change in intensity indicating the boundary between two regions in an image. It is a local property of an individual pixel and is calculated from the image function in a neighborhood of the pixel. Edge detection is a fundamental operation in computer vision and image processing. It concerns the detection of significant variations of a grey level image. The output of this operation is mainly used in higher-level visual processing like three-dimensional (3-D) reconstruction, stereo motion analysis, recognition, scene segmentation, image compression, etc. Hence, it is important for a detector to be efficient and reliable.

Edge detection has been an active research area for more than 45 years [1]. Several reviews of work on edge-detection are available in literature [2], [3], [4], [5], and [6]. Surface fitting approach for edge detection is adopted by several authors [7], [8], [9], and [10]. Bergholm's [11] edge detector applies a concept of edge focusing to find significant edges. Detectors based on some optimality criteria are developed in [12], [13], and [14]. Use of statistical procedures are illustrated in [15], [16], and [17]. Other approaches on edge detection include the use of genetic algorithms [18], and [19], neural networks [20], the Bayesian approach [21], and residual analysis-based techniques [22], and [23]. Some authors have tried to study effect of noise in images on the performance of edge detectors [24].

In general, edge detection can be classified in two categories: gradient operators and second derivative operators. Due to an edge in an image corresponds to an intensity change abruptly or discontinuity, step edge contain large first derivatives and zero crossing of the second. In the case of first derivative operation, edge can be detected as local maximum of the image convolved with a first derivative operator. Prewitt, Robert, Sobel [25] and Canny [12] implement their algorithms using this idea. For the second derivative case, edges are detected as the location where the second derivative of the image crosses zero. D. Marr and E.C. Hildreth [26] examine the zero-crossing of the Laplacian, and R. M. Haralick [8] examines the zero-crossing of the second derivative along the gradient direction. J. Clark [27] shows that zero-crossing methods can produce phantom edges, which have no correspondence to significant changes in image intensity, he provides a method for classifying zero-crossing edges as authentic or phantom edges.

This paper provides a method for localizing step edge as:

1. Zero or positive minimum of the magnitude of the second derivative along the gradient direction, it is very simple but also suffers from phantom edges and we provide a method for discard these phantom edges,
2. The negative minimum of the third derivative.

This paper is arranged as follows: Section-2 gives the mathematical description of the method. In Section-3, a comparative evaluation of the method is illustrated. Finally, a conclusion is presented in Section-4.

2 Mathematical Description of the Method

First, let us for the purposes of clarity in our exposition make the following lemmas:

Lemma 1: the first derivative operators require thinning and thresholding [29].

Lemma 2: the use of higher order derivatives makes the detector susceptible to high frequency noise and also results in poorer localization of edges [28].

Lemma 3: the contrast of an edge of a function f(x) is the magnitude of the first derivative of f(x) at the edge.

That is, $c(x) = |f_x|, (f_x = \partial f / \partial x)$ [27].

Lemma 4: an edge of the smoothed intensity function f(x) is an authentic edge if $|f_x|$ is a maximum [27].

Lemma 4: an edge of the smoothed intensity function f(x) is a phantom edge if $|f_x|$ is a minimum [27].

We start a simple model for edges, namely a step edge. Let the step edge at location z with step size s is denoted by f:

$$f(x) = s\, u\,(x-z) \tag{1}$$

Where u(x) is the unit step function.

The Gaussian low-pass filter g(x) is given by:

$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2 / 2\sigma^2} \tag{2}$$

its first derivative is given by:

$$D(x) = \frac{-x}{\sigma^3\sqrt{2\pi}} e^{-x^2 / 2\sigma^2} \tag{3}$$

Differentiation can be achieved by convolution of the signal f(x) with derivatives of this filter. The first derivative of the smoothed step is:

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$$f_x = f(x) * D(x) = \int_{-\infty}^{x} f(z) D(x-z) \partial z \quad (4)$$

$$f_x = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} su(z-z) \left[-(x-z) e^{-(x-z)^2 / 2\sigma^2} \right] \partial z \quad (5)$$

$$f_x = \frac{s}{\sigma\sqrt{2\pi}} e^{-(x-z)^2 / 2\sigma^2} \quad (6)$$

It is exactly the first derivative of a Gaussian function, in which the peak is located at z, and the amplitude is proportional to s, the size of the step.
The second derivative of the smoothed step is:

$$f_{xx} = \frac{-s}{\sigma^3\sqrt{2\pi}} (x-z) e^{-(x-z)^2 / 2\sigma^2} \quad (7)$$

its magnitude is:

$$\left| f_{xx} \right| = \frac{s}{\sigma^3\sqrt{2\pi}} (x-z) e^{-(x-z)^2 / 2\sigma^2} \quad (8)$$

The zero-crossing in f_{xx} and $|f_{xx}|$ is located at z, the peak and valley are at a distance of σ on either side of z, where there are two peaks in $|f_{xx}|$ also at a distance of σ on either side of z.
The third derivative of the f(x) is:

$$f_{xxx} = \frac{s}{\sigma^3\sqrt{2\pi}} \left(\frac{(x-z)^2}{\sigma^2} - 1 \right) e^{-(x-z)^2 / 2\sigma^2} \quad (9)$$

at x = z, f_{xxx} has a negative minimum equal to $\frac{-s}{\sigma^3\sqrt{2\pi}}$.
The first, second, and third derivatives of f(x) are depicted in figure1.

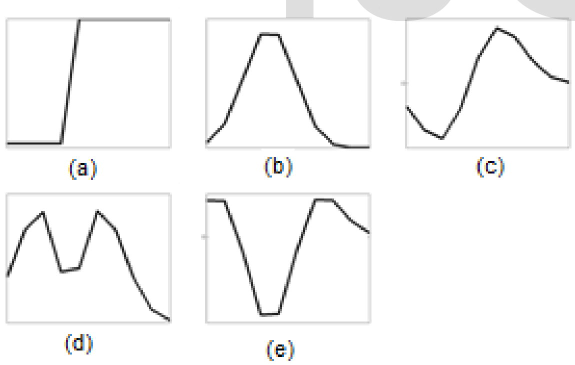

Fig. 1: (a) Step edge, (b) First derivative, (c) Second derivative, (d) Magnitude of second derivative, (e) Third derivative

From Fig. 1, the step edge can be localized as:
1. Maxima of the first derivative taken in the direction of the gradient or the gradient modulus,
2. Zero-crossings of the Laplacian or the second directional derivative along the gradient direction,
3. Zero or positive minimum, due to the lack of symmetry of the edge profile, of the magnitude of the second derivative along the gradient direction,
4. Negative minimum of the third derivative.

Prewitt, Robert, and Sobel [25], implement their algorithms using the first directional derivative. Canny [12], consider the local maxima of the gradient modulus and uses a non-maxima suppression followed by thresholding with hysteresis algorithms to obtain smooth and connected edge map. D. Marr and E.C. Hildreth [26] examine the zero-crossing of the Laplacian, and R. M. Haralick [8] examines the zero-crossing of the second derivative along the gradient direction. J. Clark [27] shows that zero-crossing methods can produce phantom edges, which have no correspondence to significant changes in image intensity, he provides a method for classifying zero-crossing edges as authentic or phantom edges.

The proposed method examines the zero or the positive minimum of the magnitude of the second derivative, and the negative minimum of the third derivative as follows:

Method 1:
In this method, the edge is localized as the zero or the positive minimum of the magnitude of the second directional derivative, so, the method is process to search for the pixels which are not zero or positive minimum and discard them from the magnitude of the first derivative of the smoothed signal (contrast, (lemma 3)). This procedure is seem to be similar to the process of non-maxima suppression, which is used in methods which localized edges as maxima in the magnitude of the first derivative of the smoothed signal to determine where the maxima occurs. The output of this process may contain phantom edges, this is due to that these zeros occur at both maxima and minima of the first derivative [27]. So, the procedure is continued to classify the edges as authentic or phantom edges. This requires the use of lemma 4, and 5. Lemma 4 defines an authentic edge as an edge for which the magnitude of the first derivative is a maximum. This equivalent to requiring the following condition be true [27]:

$$\frac{\partial f}{\partial x} \frac{\partial^3 f}{\partial x^3} < 0 \quad (10)$$

Similarly, a phantom edge is indicated if:

$$\frac{\partial f}{\partial x} \frac{\partial^3 f}{\partial x^3} > 0 \quad (11)$$

if $\frac{\partial f}{\partial x} \frac{\partial^3 f}{\partial x^3} = 0$, we say there is no edge, as either the contrast $\left|\frac{\partial f}{\partial x}\right|$ is zero or $\frac{\partial^3 f}{\partial x^3}$ is zero, in which case we do not have a zero or positive minimum of $\frac{\partial^2 f}{\partial x^2}$.

Advantages and disadvantages:
1- The advantage of this method over the gradient methods is that there is no need to use the gradient direction as in the gradient methods which may cause an error in estimating this direction. The disadvantages are that the use of higher order derivatives may results in poorer localization of edges (lemma 2), and, this method requires two thresholds, this is due to that there may be phantom edges where contrast is greater than some authentic edges, so that, while thresholding will get rid of most of the phantom edges, it will not eliminate all of them and will eliminate some of the authentic edges [27]. Note that, some of the authentic edges may be due to noise and thus may not

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correspond to significant image detail. The edge classification procedure will therefore not remove these noise edges, and some other method must be used to eliminate them, such as thresholding of the edge contrast after the classification process (lemma 1).

2- The advantage of this method over the zero-crossing methods is that the search for the zero or positive minimum is simpler than the search for the zero cross, furthermore, the use of the edge contrast makes the detector less susceptible to noise.
3- The disadvantages are the use of two thresholds.

Method 2:
In this method, the edge is localized as the negative minimum of the third derivative. The procedure is processed in two ways:
Method (2-1): Search for the pixels which are not negative minimum and discard them from the edge contrast. But, this method requires two thresholds.
Method (2-2): Search for the pixels which are not negative minimum and discard them from the third derivative image and remain the pixels which are negative minimum. But, this makes the detector more susceptible to noise than method (2-1).
Advantages and disadvantages:
The advantages and disadvantages of these methods are as in method 1.

3 Comparative Evaluation of the Proposed Edge Detectors

We begin the evaluation by comparing the output of our algorithms with those obtained using Canny [12] and Marr-Hildreth [26] algorithms for three images: Lena image (Fig. 2), X-ray image of knee (Fig. 3), and Text image (Fig. 4). The reason for choosing these methodologies for comparison is that they are considered as standard methods in edge detection.

Fig. 2: A 256 × 256 Lena image.

Fig. 3: X-ray image of knee

Fig. 4: Text image

The output of the proposed, Canny, and Marr-Hildreth edge detectors for the Lena image (Fig. 2) is shown in Fig. 5. For each algorithm, the threshold parameters are manually adjusted for best results.

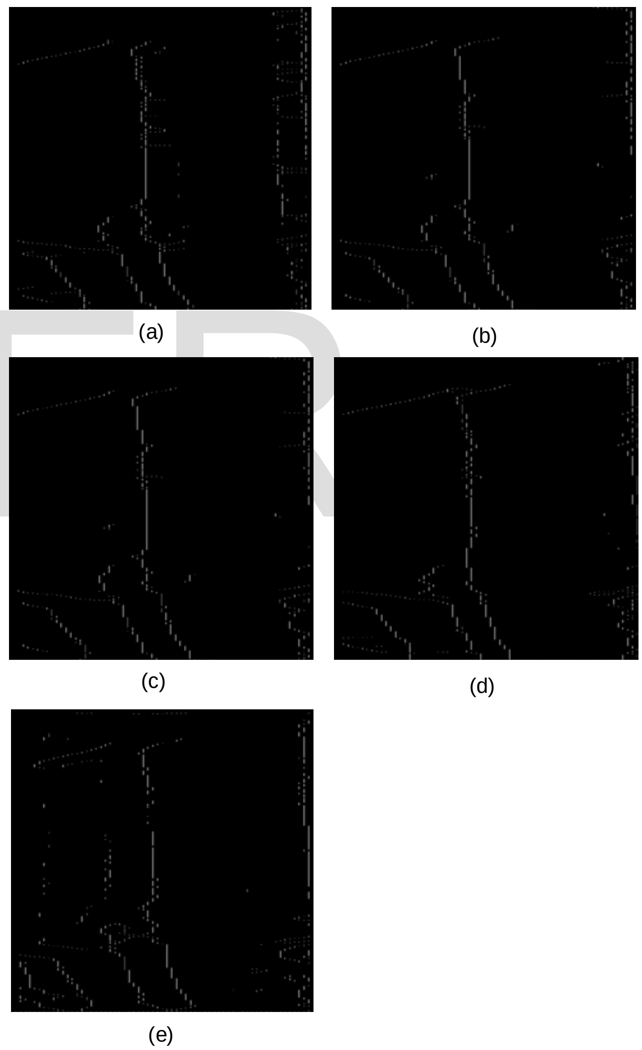

Fig. 5: Edges detected from Lena image (Fig.2). (a) using method 1, (b) using method (2-1), (c) using method (2-2), (d) using Canny method, (e) using Marr-Hildreth method.

It can be seen from Fig. 5 that many of the edges visible to the eye are detected by our algorithms.
The results obtained from the proposed algorithms, Canny, and Marr-Hildreth are now compared for the X-ray image of

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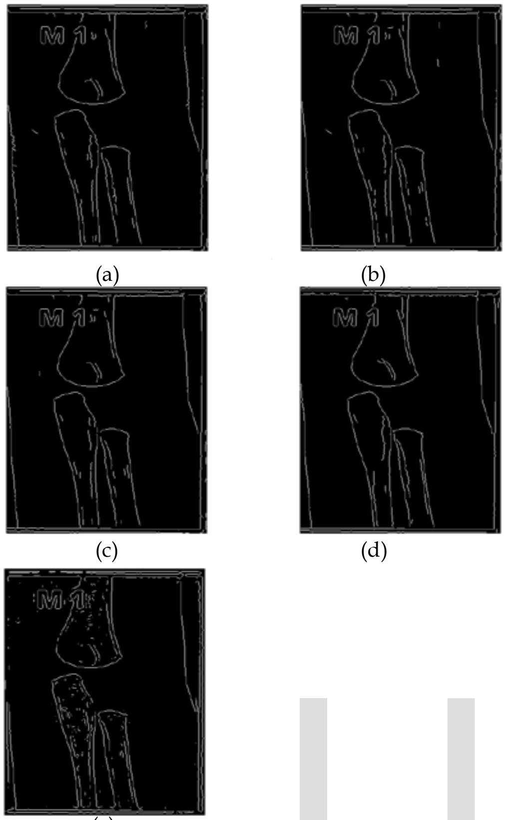
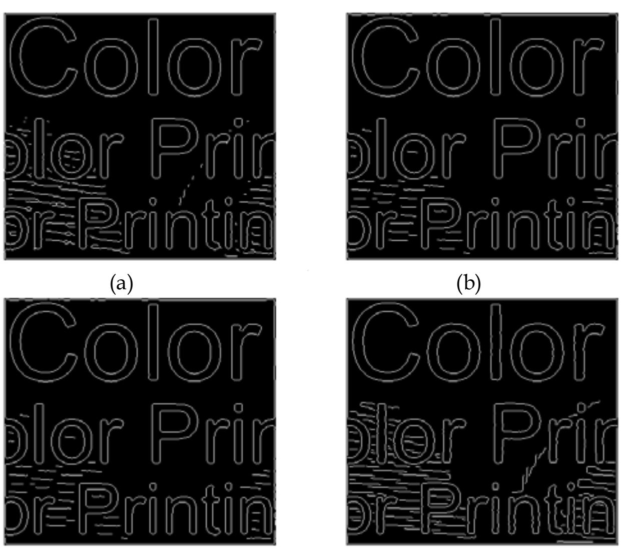
knee (Fig. 3). Edge maps of the image extracted by the algorithms are shown in Fig. 6.



(a) (b) (c) (d) (e)

Fig. 6: Edges detected from X-ray image of knee (Fig. 3). (a) using method 1, (b) using method (2-1), (c) using method (2-2), (d) using Canny method, (e) using Marr-Hildreth method.

Fig. 6 shows that our procedures produced a clean edge map free from many spurious edges.
The output of the proposed, Canny, and Marr-Hildreth edge detectors for the Text image (Fig. 4) is shown in Fig. 7.



(a) (b) (c) (d)

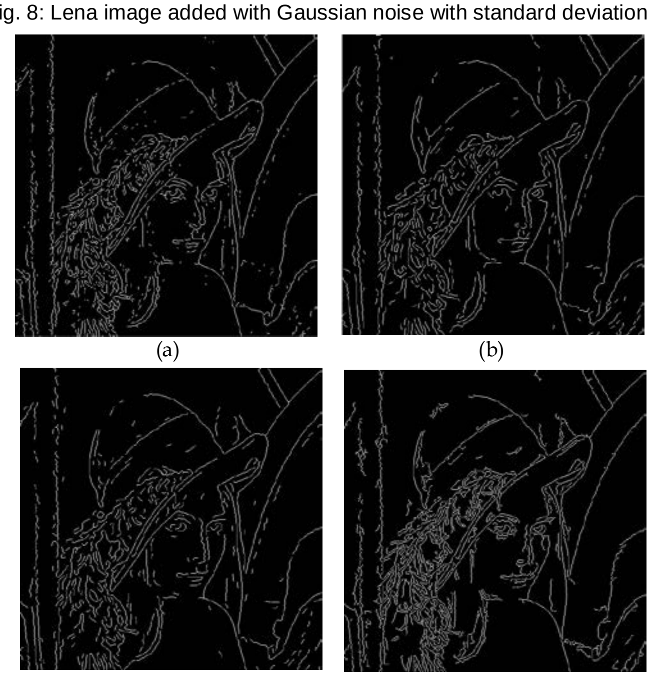
(e)

Fig. 7: Edges detected from Text image (Fig. 4). (a) using method 1, (b) using method (2-1), (c) using method (2-2), (d) using Canny method, (e) using Marr-Hildreth method.

It may be noted that a visibly nicer edge map is obtained for the Text image by our algorithms, as shown in Fig. 7(a), (b), and (c).

We now try our algorithm on a noisy image in order to judge the performance of the proposed algorithm vis a vis the other algorithms. For this purpose, the Lena image added with Gaussian noise with standard deviation 20 (Fig. 8) is considered. The outputs of the proposed algorithm and the other algorithms are shown in Fig. 9

Fig. 8: Lena image added with Gaussian noise with standard deviation 20.



(a) (b) (c) (d)

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(e)

Fig. 9: Edge detected from the corrupted Lena image (Fig. 8).
(a) using method 1, (b) using method (2-1), (c) using method (2-2),
(d) using Canny method, (e) using Marr-Hildreth method

It is clear from the figure that the proposed method produced an edge map containing many visibly important edges even for this noisy image, though the edge positions are slightly distorted. Further, there are only a few spurious edges in this edge map.

4 Conclusion

Physical edges are one of the most important properties of objects. They correspond to object boundaries or to changes in surface orientation or material properties. Edges help to extracting useful information and characteristics of an image. The majority of existing edge detectors is intended for the step edges. This is a significant limitation, because the consideration of several edge types will simplify a number of problems in artificial vision and image processing.

In this paper, the application of a new algorithm to the problem of edge detection has been discussed. The proposed method is based on the behavioral study of the step edges with respect to differentiation operators.

The results of the proposed algorithm have been compared with various methods in the preceding section. The comparison is performed on 1) real life images without noise and 2) a real life image with noise. It is to be noted that on all the real life images considered, the proposed algorithm produced fairly good results.

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